Algebraically Identifying Quadric Surfaces

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Abstract

This article discusses a logical process to quickly identify, algebraically, what quadric surface a given equation is. This method ought to be used alongside, or as a heuristic, to drawing level curves of the surface, instead of completely replacing it.

Dedicated to Andy and Pranav.

Disclaimer: We will not concern ourselves with all quadric surfaces; we will only examine a small subset of them. These were the ones that I was required to learn in my Calculus-III (MA-261) course.

For posterity, the subset of quadric surfaces in the scope of this document are defined as follows:

Definition 1: "Simple Quadric Surfaces"

$$Ax^a + By^b + Cz^c = D; \qquad a, b, c \in \{0, 1, 2\} \qquad A, B, C \in \{\mathbb{R}\}$$

A "simple" quadric surface is a linear combination of x, y *and* z *with degree either* 0, 1 *or* 2.

1. Count Variables.

    (a) If it is only one or two variables, it is a cylindrical surface.

2. Isolate the constant, and make it positive. If it's zero, go to step 4.

3. Count the number of linear terms.

    - All quadratic terms:

        (a) How many quadratic terms are negative?
            - Zero? Ellipsoid or sphere.
            - One? Hyperboloid of one sheet.
            - Two? Hyperboloid of two sheets.
            - Three? Degenerate/no solution.

    - Two quadratic terms, one linear term:

        (a) Are the quadratic terms the same sign?
            - Yes? (Elliptical) parabaloid.
            - No? Hyperbolic parabaloid.

    - One quadratic term and two linear terms? Cylindrical parabola.

    - No quadratic term and three linear terms? Plane. Not a quadric surface.

4. Count the number of quadratic terms.

    - Three:

        (a) Are all the terms the same sign?
            - Yes? A point.
            - No? A cone.

    - Two:

        (a) Are the quadratic terms the same sign?
            - Yes? (Elliptical) hyperboloid.
            - No? Hyperbolic parabaloid.

    - One? Not a simple quadric surface (it's a rotated parabolic cylinder).

    - Zero? A plane. Not a quadric surface.